Properties of Parallelograms - Notes

Parallelogram – a quadrilateral where opposite sides are parallel.

If a quadrilateral is a parallelogram, then it has all SEVEN of these characteristics.

1. If a quadrilateral is a parallelogram, then opposite sides are parallel.

   \[ \overline{AB} \parallel \overline{CD} \]
   \[ \overline{AD} \parallel \overline{BC} \]

2. If a quadrilateral is a parallelogram, then opposite sides are congruent.

   \[ \overline{AB} \cong \overline{CD} \]
   \[ \overline{AD} \cong \overline{BC} \]

3. If a quadrilateral is a parallelogram, then opposite angles are congruent.

   \[ \angle A \cong \angle C \]
   \[ \angle B \cong \angle D \]

4. If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

   \[ \angle A + \angle B = 180^\circ \]
   \[ \angle B + \angle C = 180^\circ \]
   \[ \angle C + \angle D = 180^\circ \]
   \[ \angle D + \angle A = 180^\circ \]
5. If a quadrilateral is a parallelogram, then its diagonals bisect each other.

\[ \text{AE} \cong \text{EC} \]
\[ \text{BE} \cong \text{ED} \]

6. If a quadrilateral is a parallelogram, then one pair of opposite sides are both parallel and congruent.

\[ \text{AB} \parallel \text{CD} \]
\[ \text{AB} \cong \text{CD} \]

7. If a quadrilateral is a parallelogram, each diagonal divides the quadrilateral into 2 congruent triangles. (by rotational symmetry or parallel angle relationships)

\[ \triangle \text{ACD} \cong \triangle \text{CAB} \]
\[ \triangle \text{DAB} \cong \triangle \text{BCD} \]